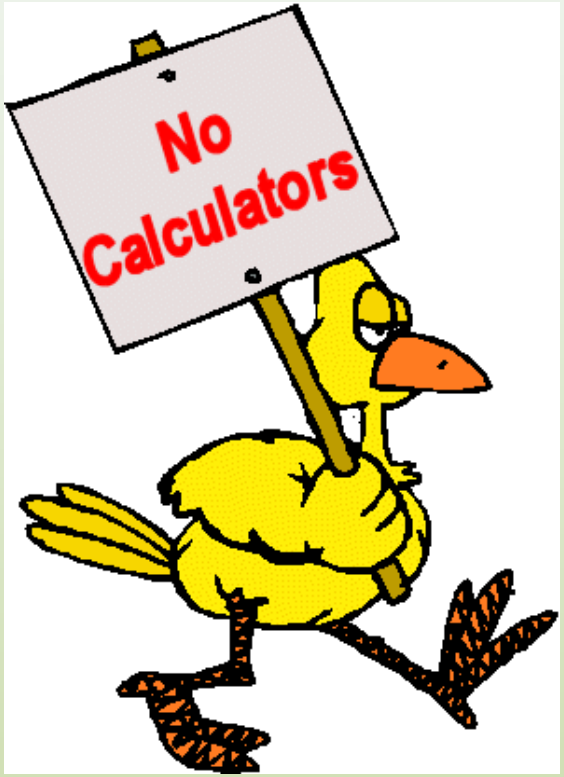
# Starter Question




Simplify these logs and work out the answers.

1. $\log_3 18 - \log_3 8 + \log_3 4$

2. $8 \log_4 2 - 6 \log_4 8$

Simplify these logs.

3. $4 \log x + 6 \log y$

4. $\frac{1}{2} \log x - 2 \log y$

# J Vectors

## J5

Use vectors to solve problems in pure mathematics and in context, including forces and kinematics.

### Assessed at AS and A-level

### Teaching guidance

Students should be able to use vectors in 2 dimensions to solve problems.

Note: at AS, questions on kinematics will **not** involve vectors but questions related to forces may involve 2D vectors.

# 6.2 Modelling with Vectors

An object's motion will have a **magnitude** and a **direction** so can be modelled using vectors:

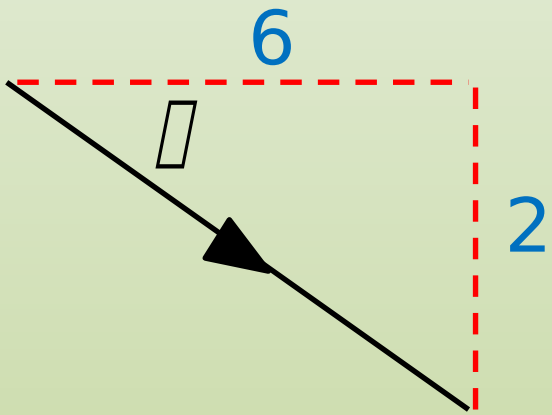
- **Displacement** is the **distance** an object has travelled in a given **direction**.
- **Velocity** is the **speed** of an object with a **direction**.
- **Acceleration** is the rate at which an object's **velocity changes**.

**Forces** can also be modelled with vectors.

# 6.2 Modelling with Vectors

**Example 1**

The acceleration of a particle is given by the vector $\mathbf{a} = (6\mathbf{i} - 2\mathbf{j})\ \text{ms}^{-2}$. Find the magnitude of the acceleration, and the angle this vector makes with the horizontal axis.

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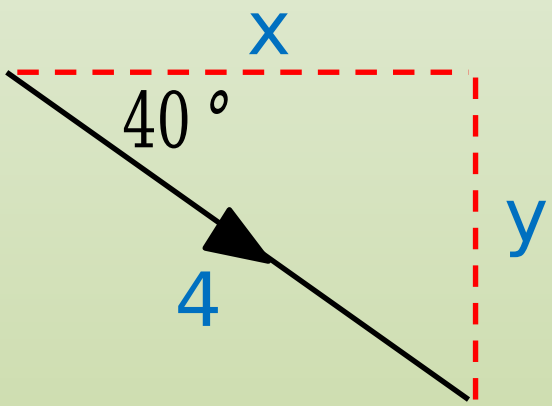
below the horizontal

# 6.2 Modelling with Vectors

## Example 2

A ball's velocity is modelled by vector $\mathbf{v} = x\mathbf{i} - y\mathbf{j}$ ms$^{-1}$, with a magnitude of 4 ms$^{-1}$ and direction of 40° below the positive $x$-axis. Find the $x$ and $y$ components of vector v. Give your answer to 4 s.f.



# 6.2 Modelling with Vectors

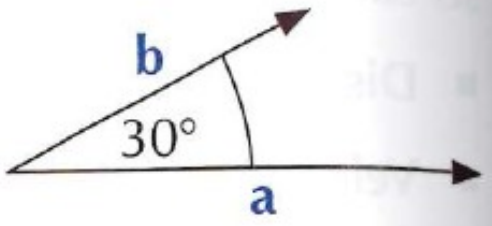
The effect of **two forces** working together can also be modelled by vectors.

These two forces will probably form a triangle without a right-angle, so you will need to use the **sine rule** and/or the **cosine rule**.

# 6.2 Modelling with Vectors

## Example 3

Two tug boats are pulling a ship with an angle of 30° between them. One tug boat exerts a force of 10 kN and is modelled with vector a. The other boat exerts a force of 12 kN and is modelled with vector b.




The resultant force on the ship, r, is the resultant vector of these two forces. Calculate the size of the resultant force.

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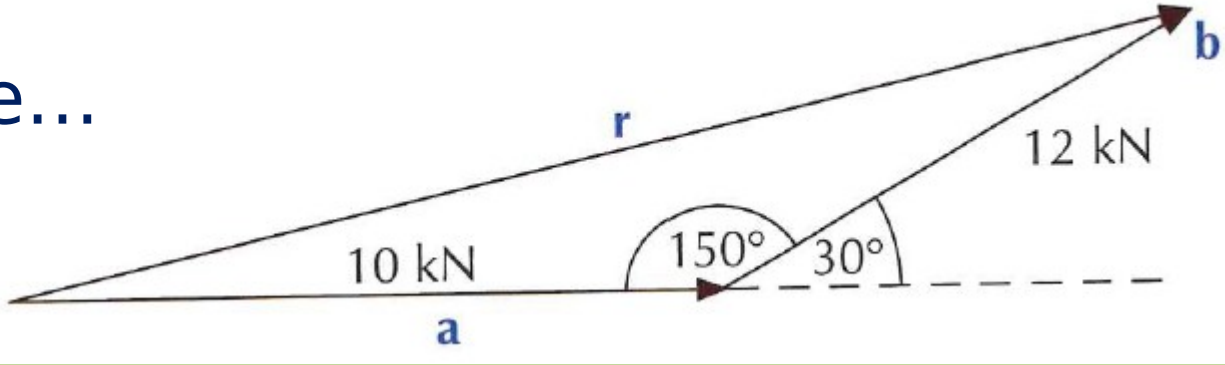
raw end to end with an angle of  between them.

se the cosine rule...



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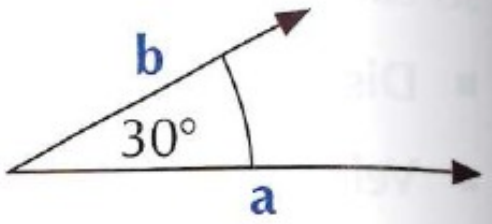


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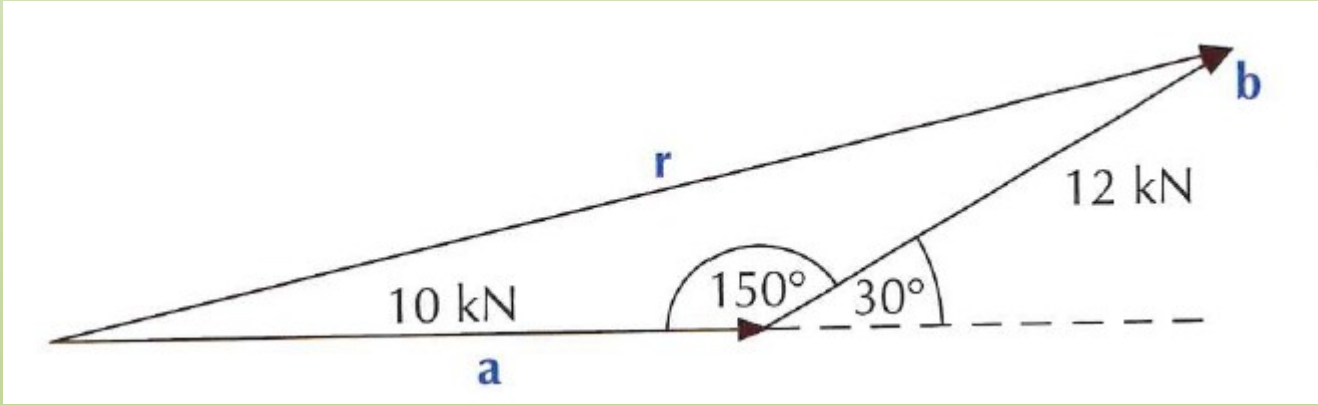


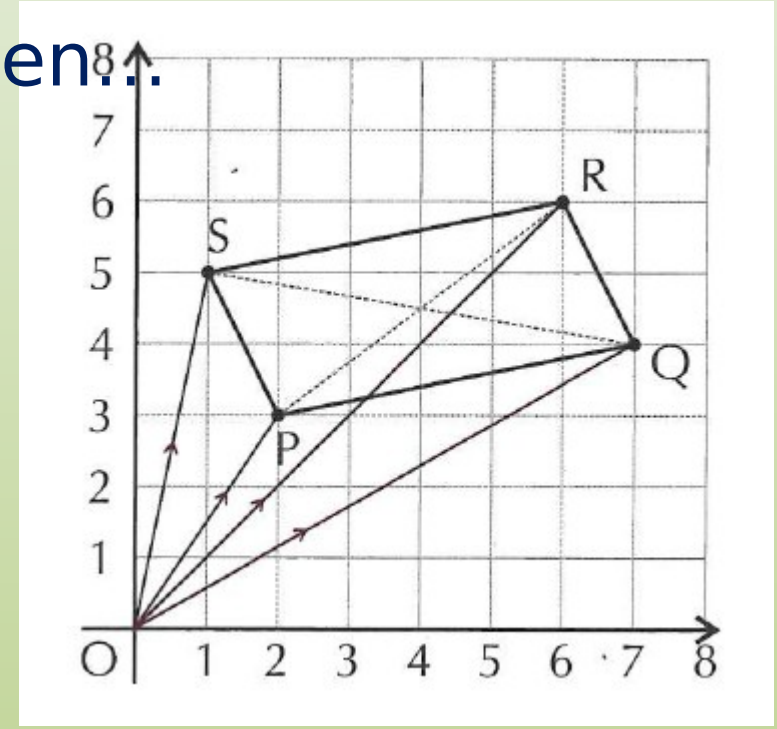
# 6.2 Modelling with Vectors

## Example 4

The position vectors of the vertices of the parallelogram PQRS are:
$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j}$, $\overrightarrow{OQ} = 7\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OR} = 6(\mathbf{i} + \mathbf{j})$ and $\overrightarrow{OS} = \mathbf{i} + 5\mathbf{j}$.
What are the exact lengths of this parallelogram's diagonals?

raw a diagram if one is not given!.

agonal PR:



# 6.2 Modelling with Vectors

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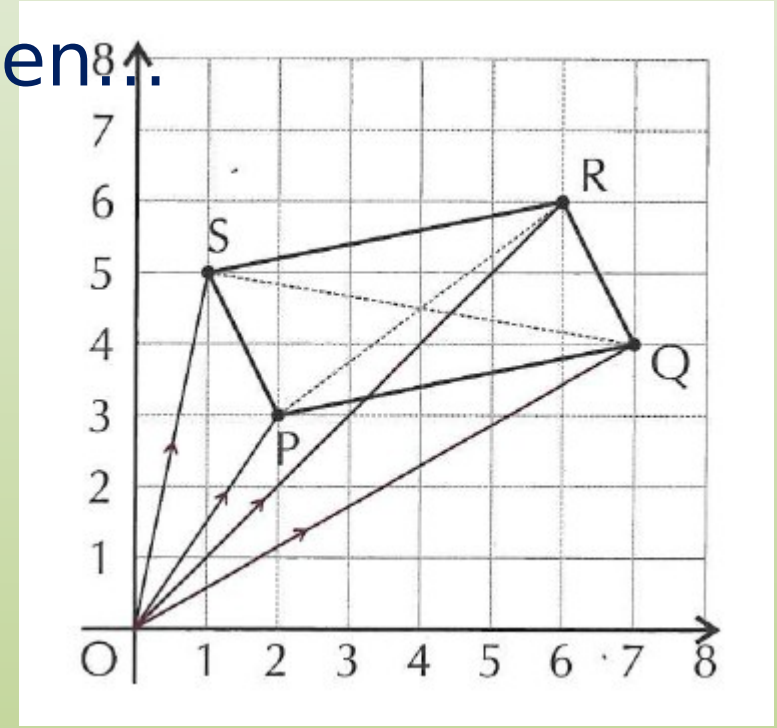


# 6.2 Modelling with Vectors

Vectors are also really useful for modelling the **direction** something is **travelling** in, e.g. the flight path of a plane.

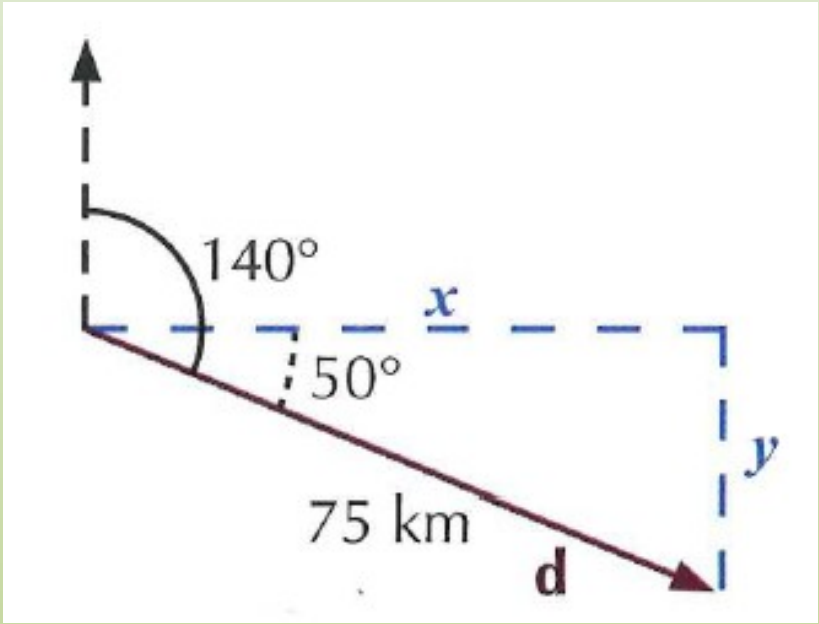
The **bearing** the vehicle travels on can be used to calculate the vector's **direction**. The **distance** it travels is the **magnitude** of its **displacement** vector, and its **speed** is the magnitude of its **velocity** vector.

# 6.2 Modelling with Vectors

**Example 5**

A ship travels 75 km on a bearing of 140°. The ship's displacement is modelled by the vector $\mathbf{d} = \begin{pmatrix} x \\ y \end{pmatrix}$. Calculate $x$ and $y$ (to 2 d.p.).

Draw a diagram!!




**CGP Ex 3.1**